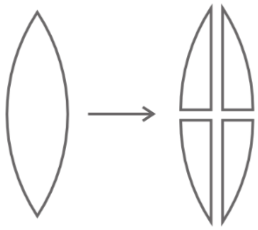
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. An equiconvex lens of focal length f, is cut into four parts as shown in the diagram. The focal length of each part is



(1) f (2) $2f$

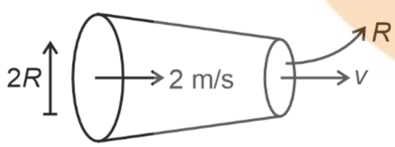
(3) $f/2$ (4) $4f$

Answer (2)

Sol. $f = \dfrac{R}{2(\mu - 1)}$ (equiconvex lens)

$f' = \dfrac{R}{(\mu - 1)}$ (plans convex lens)

$f' = 2f$

2. 

Radius of a tube decreases from $2R$ to R in which ideal liquid is flowing at same level.

Speed at one end is 2 m/s as shown, find speed v at other end

(1) 4 m/s (2) 1 m/s

(3) 2 m/s (4) 8 m/s

Answer (4)

Sol. $A_1v_1 = A_2 = v_2$ equation of continuity

$\pi(2R)^2 \cdot 2 = \pi R^2 v$

$v = 8$ m/s

3. The dimensional formula of capacitance is

(1) $[M^{-1}L^2T^2A^{-3}]$ (2) $[M^{-1}L^{-2}T^4A^3]$

(3) $[M^{-1}L^{-2}T^4A^2]$ (4) $[M^{-1}L^{-2}T^2A^2]$

Answer (3)

Sol. The energy stored in capacitor in term of charge

$$E = \frac{Q^2}{2C}$$

$$C = \frac{Q^2}{2E}$$

$$[C] = \frac{[A^2T^2]}{[ML^2T^{-2}]}$$

$$= [M^{-1}L^{-2}T^4A^2]$$

4. A proton is moving with uniform velocity of 2×10^8 m/s in uniform magnetic and electric fields which are perpendicular to each other. If electric field is switched off then proton moves in circular path of radius 1.6×10^{-5} m. Then magnetic field is B

(1) 5×10^{-5} T (2) 1.2×10^5 T

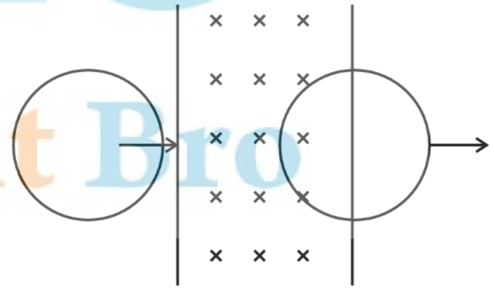
(3) 2.5×10^4 T (4) 2.5×10^2 T

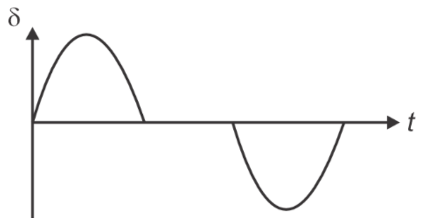
Answer (2)

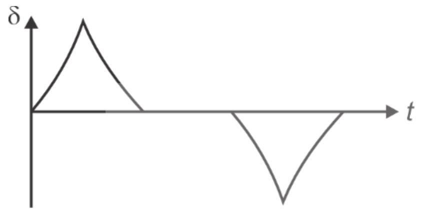
Sol. $r = \dfrac{mv}{qB} \Rightarrow 1.6 \times 10^{-5} = \dfrac{1.6 \times 10^{-27} \times 2 \times 10^8}{1.6 \times 10^{-19} \times B}$

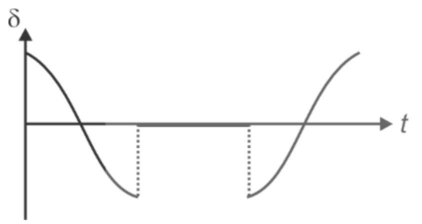
$$B = \frac{5}{4} \times 10^5 = 1.25 \times 10^5 \text{ T}$$

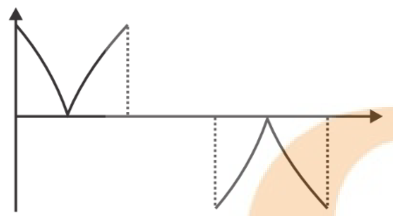
5. A conducting circular ring is moving with a constant velocity in a uniform magnetic field as shown. Identify the correct graph between induced emf vs time



(1) 

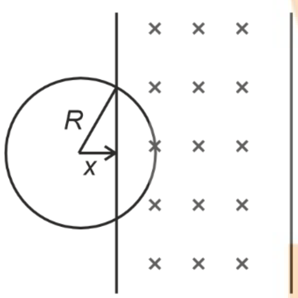
(2) 

(3) 

(4) 

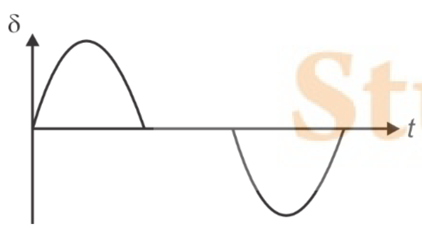
Answer (1)

Sol.



$\delta = 2Bv\sqrt{R^2 - x^2}$ (Equation of an ellipse)

$x = R - vt$



6. The displacement of a particle moving under the action of a force $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$ is $\vec{d} = \hat{i} + \hat{j} + \hat{k}$. Find the value of b if the work done by the force is zero.

(1) 0 (2) +3

(3) –3 (4) –1

Answer (3)

Sol. Work = $\vec{F}\cdot\vec{s} = (2\hat{i} + b\hat{j} + \hat{k})\cdot(\hat{i} + \hat{j} + \hat{k})$

$= 2 + 1 + 1 = 3 + b = 0$

$\Rightarrow b = -3$

7. In a series LCR circuit the maximum amplitude of current is I_0 when the resistance is R. What is the maximum amplitude of current if the resistor is replaced by a resistor of resistance $\frac{R}{2}$.

(1) I_0 (2) $2I_0$

(3) $\frac{I_0}{2}$ (4) $\frac{2I_0}{3}$

Answer (2)

Sol. Current has maximum amplitude at resonance.

$$I_0 = \frac{\xi_0}{R}$$

$$\Rightarrow I_0' = \frac{\xi_0}{R/2} = \frac{2\xi_0}{R} = 2I_0$$

8. **Statement-I :** Fringe width of red light is more than fringe width of violet light.

Statement-II : Fringe width is directly proportional to the wavelength of light used.

Choose the correct option.

(1) Statement-I is correct and statement-II is incorrect

(2) Both statement-I and statement-II are correct

(3) Statement-I is incorrect and statement-II is correct

(4) Both statement-I and statement-II are incorrect

Answer (2)

Sol. Fringe width $(\beta) = \frac{\lambda D}{d}$

9. For non-vibrating diatomic gas has adiabatic constant of γ_1 & for vibrating diatomic gas has adiabatic constant of γ_2 then

(1) $\gamma_1 > \gamma_2$ (2) $\gamma_1 < \gamma_2$

(3) $\gamma_1 = \gamma_2$ (4) None of these

Answer (1)

Sol. $\gamma_1 = 1 + \frac{2}{5} = \frac{7}{5} = 1.4$

$\gamma_2 = 1 + \frac{2}{7} = \frac{9}{7} = 1.28$

Therefore $\gamma_1 > \gamma_2$

10. A force $\vec{F} = (\hat{i} + 2\hat{j} - 3\hat{k})N$ acts on point whose position vector is given as $\vec{r} = (2\hat{i} - 3\hat{j} + 7\hat{k})m$. Find torque about origin.

(1) $(+5\hat{i} - 12\hat{j} + 7\hat{k})N.m$

(2) $(-5\hat{i} - 12\hat{j} + 8\hat{k})N.m$

(3) $(-5\hat{i} + 13\hat{j} + 7\hat{k})N.m$

(4) $(-5\hat{i} + 13\hat{j} - 7\hat{k})N.m$

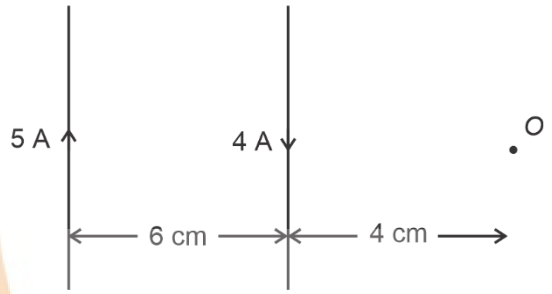
Answer (3)

Sol. $\vec{T} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 7 \\ 1 & 2 & -3 \end{vmatrix}$

$= \hat{i}(9 - 14) - \hat{j}(-6 - 7) + \hat{k}(4 + 3)$

$= (-5\hat{i} + 13\hat{j} + 7\hat{k})\ N.m$

11. The net magnetic field at point O due to the two infinite current carrying wires shown in the figure is



(1) 1×10^{-5} T

(2) 1.2×10^{-5} T

(3) 1.5×10^{-5} T

(4) 2×10^{-5} T

Answer (1)

Sol. $B_{net} = \left| \frac{\mu_0 (5\text{ A})}{2\pi(10\text{ cm})} - \frac{\mu_0 (4\text{ A})}{2\pi(4\text{ cm})} \right|$

$= \frac{25\mu_0}{\pi} = 10^{-5}$ T

12. Read the statements and select the correct option.

Statement I : A pendulum is taken from Earth to another planet having mass four times and radius double than earth, then time period of pendulum remain same as on earth.

Statement II : The time period of pendulum only depends on the gravity of the planet.

(1) Statement I is true but statement II is false.

(2) Statement II is true but statement I is false.

(3) Both statements are false.

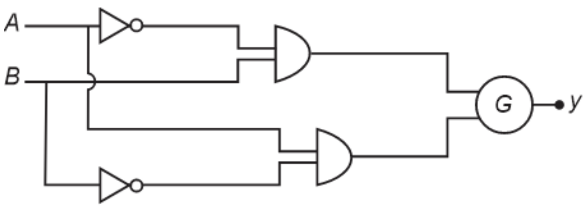
(4) Both statements are true.

Answer (1)

Sol. On earth $T = 2\pi\sqrt{\frac{l}{g}}$

On other planet, $g' = \frac{G(4M)}{(2R)^2} = g$

So, time period will remain same and her T depends on g as well as l.

13. 

For a given logic circuit truth table is given identify the gate G.

A	B	y
0	0	1
1	0	0
0	1	0
1	1	1

(1) AND (2) NOR

(3) NAND (4) OR

Answer (2)

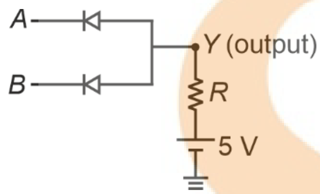
Sol. From truth table

we know its XNOR gate

i.e. $y = \overline{\bar{A}B + A\bar{B}}$

therefore gate G must be NOR gate.

14. Name the logic gate



(1) OR (2) AND

(3) NOT (4) NAND

Answer (2)

Sol. If both A and B are high only then Y is high, otherwise Y is low.

$\therefore$ AND Gate.

15. Displacement current in capacitor of area 16 cm^2 is 6 A at an instant. Find displacement current across area 3.2 cm^2

(1) 1.2 A (2) 1.6 A

(3) 2.1 A (4) 0.5 A

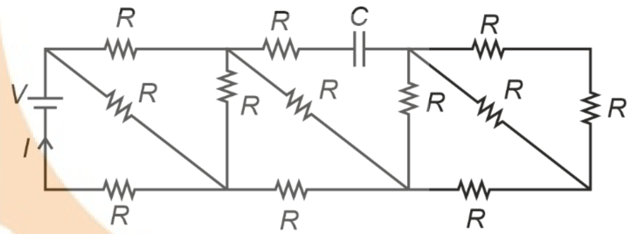
Answer (1)

Sol. $id = \epsilon_0 \dfrac{d\phi_E}{dt} = \epsilon_0 A \dfrac{dE}{dt} \Rightarrow id \propto A$

$\Rightarrow i' = \left(\dfrac{3.2}{16}\right) id$

$= 1.2$ A

16. In the RC circuit shown, find I.



(1) $\dfrac{V}{5R}$ (2) $\dfrac{5V}{3R}$

(3) $\dfrac{8V}{13R}$ (4) $\dfrac{3V}{R}$

Answer (3)

Sol. At stead state, C behaves as open-circuit

$R_{eq} = \dfrac{13}{8}R$

$I = \dfrac{V}{R_{eq}} = \dfrac{8V}{13R}$

17. A glass slab of refractive index $\mu_g = 1.44$ is coated with a thin film of refractive index $\mu_f = 2$. The minimum thickness of the film so that maximum transmission of green light of wavelength $\lambda = 5000$ Å (incident normally) takes place is

(1) 0.500 μm (2) 0.250 μm

(3) 0.125 μm (4) 1.00 μm

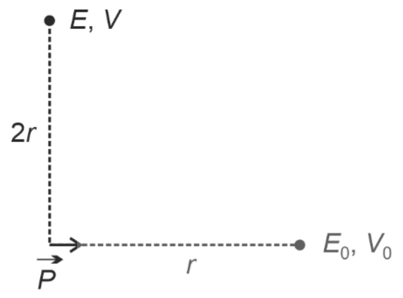
Answer (3)

Sol. For maximum transmission of light incident normally

$2\mu_f t = n\lambda$ (n = 1, 2, 3 ....)

$t_{min} = \dfrac{\lambda}{2\mu_f} = \dfrac{5000 \times 10^{-10}}{2(2)}$ m = 0.125 μm

18. For the electric dipole shown in the figure, the electric field and the electric potential are E_0, V_0 at a distance r on the axis. Then what is the electric field and the electric potential at a point on the equatorial plane at a distance $2r$.



(1) $\frac{E_0}{16}$, 0

(2) $\frac{E_0}{4}$, 0

(3) E_0, V_0

(4) $\frac{E_0}{8}$, 0

Answer (1)

Sol. $E_{axis} = \frac{2kP}{r^3} = E_0$

$$E_{equatorial} = \frac{kP}{(2r)^3} = \frac{kP}{8r^3} = \frac{E_0}{16}$$

$$V = \frac{kP\cos\theta}{r^2}$$

$$V_{axis} = \frac{kP\cos 0^\circ}{r^2} = \frac{kP}{r^2}$$

$$V_{equatorial} = \frac{kP\cos 90^\circ}{(2r)^3} = 0$$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Choose the correct answer:

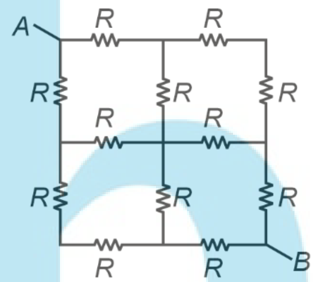
21. A projectile is fired with speed of 20 m/s at angle of 60° with horizontal. The speed at highest point of trajectory is x m/s then x is

Answer (10)

Sol. $V_H = 4\cos\theta$

$= 20\cos 60^\circ = 10$ m/s

22. If equivalent resistance across AB is $\frac{NR}{2}$, find N



Answer (3)

Sol. Line of symmetry problem

$$R_{eq} = \frac{3R}{4} \times 2 = \frac{3R}{2}$$

23.

24.

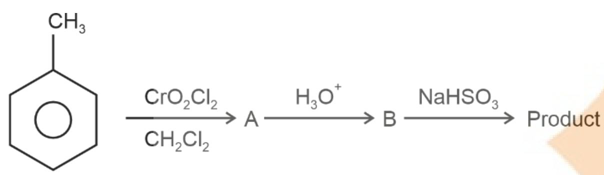
25.

CHEMISTRY

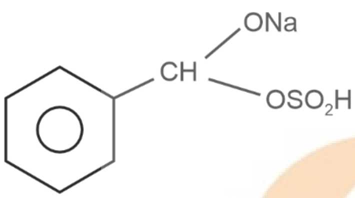
SECTION - A

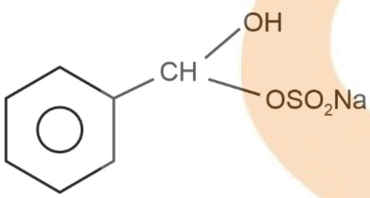
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

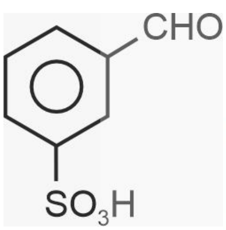
Choose the correct answer :

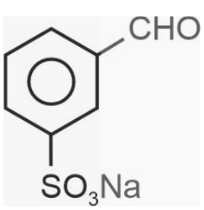
1. 

The product is

(1) 

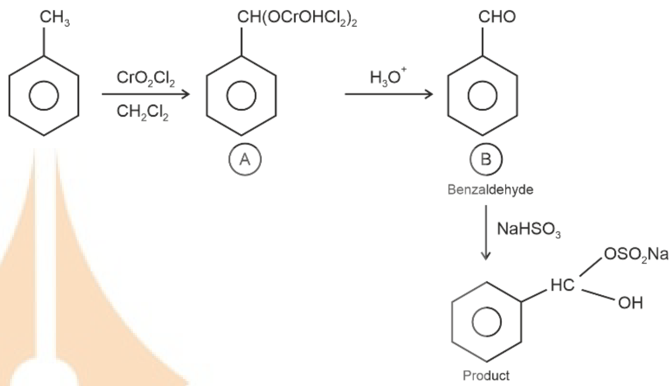
(2) 

(3) 

(4) 

Answer (2)

Sol.



2. Density of 3 M NaOH is 1.25 g/ml. Molality of solution is

(1) 2.65 (2) 2.5

(3) 2.8 (4) 3

Answer (1)

Sol. $M = \frac{1000\ M}{1000\ d - M \times M_0}$

$= \frac{3 \times 1000}{1250 - 3 \times 40}$

$= \frac{3 \times 1000}{1130} = 2.65$

3. Arrange according to CFSE.

(i) $[Co(NH_3)_4]^{2+}$

(ii) $[Co(NH_3)_6]^{3+}$

(iii) $[Co(NH_3)_6]^{2+}$

(iv) $[Co(en)_3]^{3+}$

(1) (iv) > (ii) > (iii) > (i)

(2) (iv) > (iii) > (ii) > (i)

(3) (i) > (iii) > (ii) > (iv)

(4) (i) > (ii) > (iii) > (iv)

Answer (1)

Sol. Crystal Field Splitting Energy (CFSE)

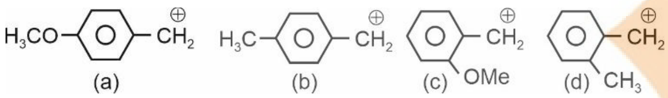
$\propto$ Charge on central metal ion

$\propto$ Ligand field strength

On this basis the correct decreasing order of CFSE

$[Co(en)_3]^{3+} > [Co(NH_3)_6]^{3+} > [Co(NH_3)_6]^{2+} > [Co(NH_3)_4]^{2+}$

4. What is correct order of stability of carbocation.



(1) a > b > c > d

(2) c > a > d > b

(3) a > c > d > b

(4) c > b > a > d

Answer (3)

Sol. Solution stability of $C^{\oplus} \propto +M,\ HC,\ +I$

$\propto \dfrac{1}{-M,\ -Z}$

5. Which of the following anion will not undergoe disproportionation?

(1) ClO_4^- (2) ClO_3^-

(3) ClO_2^- (4) ClO^-

Answer (1)

Sol. In $ClO_4^- \rightarrow$ chlorine is in its highest oxidation state i.e., +7.

Chlorine can exhibit –1 to +7 oxidation state.

The oxidation states of chlorine which can undergo disproportionation are : 0, +1, +3 and +5.

6. Compare dipole moment of

(I) NF_3 (II) $CHCl_3$

(III) H_2S (IV) HBr

(1) I > II > III > IV (2) II > III > I > IV

(3) II > III > IV > I (4) III > I > IV > II

Answer (3)

Sol.	NH_3	$CHCl_3$	H_2S	HBr
$\Rightarrow$	0.230	1.04	0.95	0.79

So, order is

$CHCl_3 > H_2S > HBr > NF_3$

7. Given below are two statements

S-I: Lassaigne test is used for detection of Nitrogen, phosphorous, sulphur and Halogens.

S-II: Lassaigne extract is made with magnesium metal.

(1) Both S-I and S-II are correct.

(2) Both S-I and S-II are incorrect.

(3) S-I is correct but S-II is incorrect.

(4) S-I is incorrect but S-II is correct

Answer (3)

Sol. Lassaigne extract is made with sodium metal, and not with magnesium metal.

8. Which one has two secondary Hydrogen atoms?

(1) 2, 2, 4, 4-tetramethylheptane

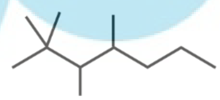
(2) 2, 2, 3, 4-tetramethylheptane

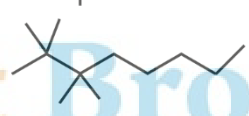
(3) 2, 2, 3, 3-tetramethyloctane

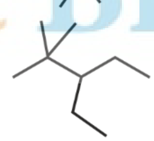
(4) 3-ethyl-2, 2-dimethylpentane

Answer (4)

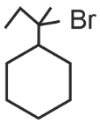
Sol. (1)  Number of 2°H = 6

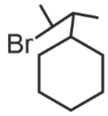
(2)  Number of 2°H = 4

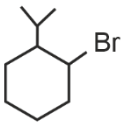
(3)  Number of 2°H = 8

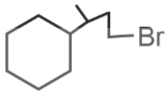
(4)  Number of 2°H = 2

9. Secondary butyl cyclohexane when reacts with Br_2 in presence of sunlight produce

(1) 

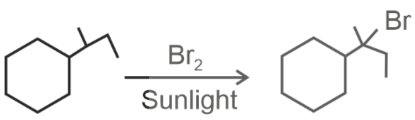
(2) 

(3) 

(4) 

Answer (1)

Sol.



10. 200 mL of 0.2 M solution of NaOH is mixed with 400 mL of 0.5 M NaOH solution. Molarity of mixture is

(1) 0.4
(2) 0.6
(3) 4 M
(4) 0.8 M

Answer (1)

Sol. $M_1 = 0.2$ M

$M_2 = 0.5$ M

$V_1 = 200$ mL

$V_2 = 400$ mL

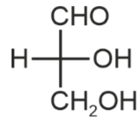
Molarity of mixture $= \dfrac{M_1V_1 + M_2V_2}{V_1 + V_2}$

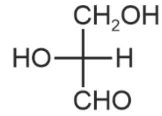
$= \dfrac{0.2 \times 200 + 0.5 \times 400}{600}$ M

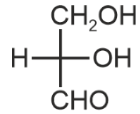
$= \dfrac{40 + 200}{600}$

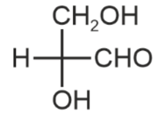
$= \dfrac{240}{600} = 0.4$ M

11. Correct structure of L-Glyceraldehyde is

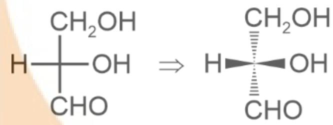
(1) 

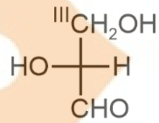
(2) 

(3) 

(4) 

Answer (3)

Sol. 



L-Glyceraldehyde

12. Identify the extensive and intensive property?

(1) Mass, volume, conductivity – Intensive property

(2) Mass, temperature, heat, volume – Extensive property

(3) Mass, volume, internal energy – Extensive property

(4) Density, temperature, moles, internal energy – Intensive property

Answer (3)

Sol. The properties which do not depend upon amount of substance is known as intensive property and the properties which depend upon amount of substance are extensive properties of matter.

Mass, volume and internal energy depend on amount of substance.

13. Among Group-15 elements, what is the maximum covalency of an element having weakest E–E bond (E = element)

(1) 4
(2) 3
(3) 5
(4) 2

Answer (3)

Sol. The E – E bond energies of the elements of group-15 are

N – N	167 kJ mol^{-1}
P – P	201 kJ mol^{-1}
As – As	146 kJ mol^{-1}
Sb – Sb	121 kJ mol^{-1}

Antimony (Sb) has the weakest E – E bond and its maximum covalency is 5.

14. What is the relation between K_{sp} and S of $Zr_3(PO_4)_4$

(1) $S=\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$ (2) $S=\left(\frac{K_{sp}}{144}\right)^{\frac{1}{7}}$

(3) $S=\frac{K_{sp}}{6912}$ (4) None

Answer (1)

Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons \underset{3s}{3Zr^{4+}}(aq) + \underset{4s}{4PO_4^{3-}}(aq)$

$K_{sp} = (3s)^3(4s)^4$

$= 27 \times 256\ S^7$

$K_{sp} = 6912\ S^7$

$S=\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

15. Match the column and choose the correct option

(A)	$\left(\frac{\partial H}{\partial T}\right)_P$	(P)	C_P
(B)	$\left(\frac{\partial G}{\partial P}\right)_T$	(Q)	C_V
(C)	$\left(\frac{\partial U}{\partial T}\right)_V$	(R)	–S
(D)	$\left(\frac{\partial G}{\partial T}\right)_P$	(S)	V

(1) (A) – (P), (B) – (S), (C) – (Q), (D) – (R)

(2) (A) – (P), (B) – (S), (C) – (R), (D) – (Q)

(3) (A) – (P), (B) – (R), (C) – (Q), (D) – (S)

(4) (A) – (Q), (B) – (S), (C) – (P), (D) – (R)

Answer (1)

Sol. Heat exchanged at constant pressure is ΔH

Heat exchanged at constant volume is ΔU

16. Consider the following statements S-1 and S-2 and choose the correct option.

S-1 : During corrosion pure metal acts as anode and impure metal acts as cathode.

S-2 : Rate of corrosion is more in alkaline medium than in acidic medium.

(1) Both S-1 and S-2 are correct

(2) Both S-1 and S-2 are incorrect

(3) S-1 is correct but S-2 is incorrect

(4) S-1 is incorrect but S-2 is correct

Answer (2)

Sol. In corrosion, a metal is oxidised by loss of electrons to oxygen. Electron released at anodic spot move through the same metal and go to another spot on the metal and reduce oxygen in the presence of H^+ (which is believed to be available from H_2CO_3 formed due to dissolution of CO_2 from air into water.

Cathode : $O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O(l)$

$E^0_{H^+|O_2|H_2O} = 1.23\ V$

Anode : $2Fe(s) \rightarrow 2Fe^{2+} + 4e^-$ $E^0_{Fe^{2+}|Fe} = -0.44\ V$

$\therefore$ Both the statements S-1 and S-2 are incorrect.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In Ru and Nb, if in Ru, 4*d* electrons are x and in Nb, 4*d* electrons are y then find the sum of x and y.

Answer (11)

Sol. $_{44}Ru \Rightarrow [Kr]\ 4d^7 5s^1$

$x = 7$

$_{41}Nb \Rightarrow [Kr]\ 4d^4 5s^1$

$y = 4$

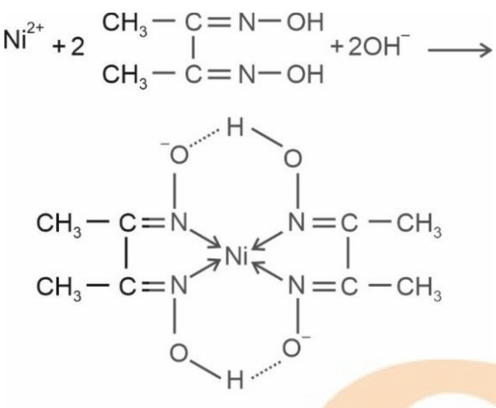
$x + y = 11$

22. $Ni^{2+} + 2DMG \longrightarrow Complex$

How many hydrogen bonds are present in a molecule of the complex?

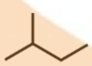
Answer (2)

Sol.



$[Ni(DMG)_2]$

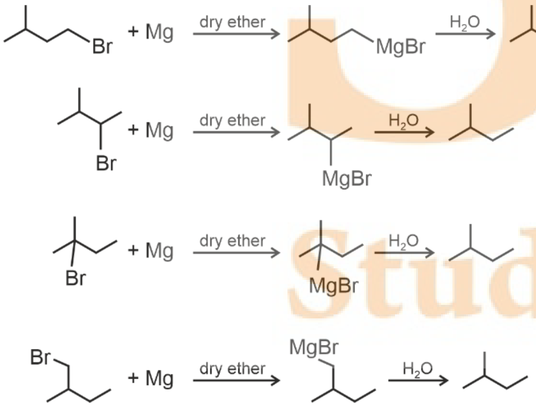
Number of H-bonds in a molecule of $[Ni(DMG)_2]$ = 2

23. $R - Br + Mg \xrightarrow{\text{dry ether}} A \xrightarrow{H_2O}$ 

How many R – Br can form isopentane?

Answer (4)

Sol.



∴ Total 4 R–Br can form isopentane in this reaction.

24. $C_6H_6 \xrightarrow{\text{Monobromination}}$ Single product

$C_6H_6 \xrightarrow[\text{Cat.}]{\text{4 moles of } H_2}$ Completely hydrogenated product

Find the number of π-electrons in C_6H_6.

Answer (8)

Sol. Degree of unsaturation of $C_6H_6 = 4$

C_6H_6 is a symmetrical dialkyne.

$\underset{(C_6H_6)}{HC \equiv C - CH_2 - CH_2 - C \equiv CH} \xrightarrow{\text{Monobromination}} \underset{\text{(Single product)}}{HC \equiv C - CH(Br) - CH_2 - C \equiv CH}$

$\underset{(C_6H_6)}{HC \equiv C - CH_2 - CH_2 - C \equiv CH} + 4H_2 \longrightarrow CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - CH_3$

Number of π-electrons in $C_6H_6 = 8$

25. Calculate the radius of first excited state of He^+ ion (in Å)

Answer (1)

Sol. $r = a_0 \dfrac{n^2}{z}$

$n = 2$

$z = 2$

$r = a_0 \dfrac{4}{2}$

$= 2a_0$

$= 2 \times 0.529$

$= 1.058$

≈ 1

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $2x^2 + (\cos\theta)x - 1 = 0$, $\theta \in [0, 2\pi]$ has roots α and β. Then the sum of maximum and minimum value of $\alpha^4 + \beta^4$.

(1) $\frac{25}{16}$

(2) $\frac{9}{16}$

(3) $\frac{41}{16}$

(4) $\frac{8}{17}$

Answer (1)

Sol. $\alpha+\beta = \frac{-\cos\theta}{2}$

$\alpha\beta = \frac{-1}{2} \Rightarrow \alpha^2\beta^2 = \frac{1}{4}$

$\Rightarrow \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{\cos^2\theta+1}{4}$

$\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$= \left(\frac{\cos^2\theta+1}{4}\right) - \frac{1}{2}$

For minimum, $\cos\theta = 0$

For maximum, $\cos\theta = 1$

$\Rightarrow$ Minimum $= 1 - \frac{1}{2} = \frac{1}{2}$

Maximum $= \frac{25}{16} - \frac{1}{2} = \frac{17}{16}$

$\Rightarrow$ Sum $= \frac{1}{2} + \frac{17}{16} = \frac{25}{16}$

2. If $\theta \in [0, 2\pi]$ satisfying the system of equations $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$. Then the sum of all real values of θ is

(1) $\frac{3\pi}{2}$

(2) π

(3) $\frac{\pi}{2}$

(4) $\frac{5\pi}{6}$

Answer (2)

Sol. $2\sin^2\theta = \cos 2\theta$

$2\cos^2\theta = 3\sin\theta$

$\Rightarrow$ Adding,

$2 = 1 - 2\sin^2\theta + 3\sin\theta$

$\Rightarrow 2\sin^2\theta - 3\sin\theta + 1 = 0$

$2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$

$2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$

$\sin\theta = 1, \frac{1}{2}$ but $2\sin^2\theta = \cos 2\theta = 2$

but not is not possible

$\Rightarrow \theta = \frac{\pi}{6}, \left(\pi - \frac{\pi}{6}\right)$

$\Rightarrow$ Sum of all values $= \frac{\pi}{6} + \pi - \frac{\pi}{6} = \pi$

3. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$.

If $f : A \to B$, number of many-one functions from A to B are

(1) 24

(2) 232

(3) 256

(4) 252

Answer (2)

Sol. $n(A) = 4$

$n(B) = 4$

Number of many one functions =

Total functions – Number of one-one function

$= 4^4 - 41 = 232$

4. 4 boys and 3 girls are to be seated in a row such that all girls seat together and two particular boys B_1 and B_2 are not adjacent to each other. Then the number of ways in which this arrangement can be done.

(1) 432 (2) 430

(3) 516 (4) 1002

Answer (1)

Sol. 4 boys and 3 girls.

All girls together $\Rightarrow$ $\boxed{G_1 G_2 G_3}$

____

3 girls and 2 boys can be seated in 3!·3! ways

Now B_1 and B_2 go into spaces.

$\Rightarrow {}^4C_2 \times 2! \times 3! \times 3! = 432$

5. If the sum $\sum_{r=0}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha.2^{29}$, then α is equal to

(1) 225 (2) 465

(3) 345 (4) 425

Answer (2)

Sol. $\frac{r^2 \cdot 30!}{(30-r)!r!} \cdot \frac{30!}{(30-r)!r!} \times \frac{(r-1)!(31-r)!}{30!}$

$= \frac{30!(31-r)}{(r-1)!(30-r)!}$

$= \frac{30(31-r)29!}{(r-1)!(30-r)!}$

$\Rightarrow \sum_{r=0}^{30} \frac{r^2({}^{30}C_r)^2}{{}^{30}C_{r-1}} = 30\sum_{r=0}^{30}(31-r)^{29}C_{r-1} = 30\sum_{r=0}^{30}(31-r)^{29}C_{30-r}$

$= 30\sum_{r=0}^{30}[(30-r)+1]^{29}C_{30-r}$

$= 30\sum_{r=0}^{30}\frac{29}{(30-r)}(30-r).^{28}C_{29-r} + 30\sum_{r=0}^{30}{}^{29}C_{30-r}$

$= 30 \cdot 29 \cdot 2^{28} + 30 \cdot 2^{29}$

$= 30 \cdot 2^{28}(29 + 2) = (31 \times 15) \cdot 2^{29}$

6. Consider a function $f(x) = \int_0^{x^2} \frac{t^2 - 8t + 15}{e^t} dt$. The number of points of extrema are

(1) 3 (2) 5

(3) 7 (4) 9

Answer (2)

Sol. $\because \quad f(x) = \int_0^{x^2} \frac{t^2 - 8t + 15}{e^t}$

$f'(x) = \frac{2x(x^4 - 8x^2 + 15)}{e^{x^2}}$

$= \frac{2x(x^2-5)(x^2-3)}{e^{x^2}}$

The extremum value of $f(x)$ are $x = 0, \pm\sqrt{5}, \pm\sqrt{3}$

$\therefore$ Number of extremum points are 5.

7. Let A and B are two events such that $P(A \cap B) = \frac{1}{10}$ and $P(A/B)$ and $P(B/A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}$ is equal to

(1) $\frac{4}{9}$ (2) $\frac{9}{4}$

(3) $\frac{3}{2}$ (4) $\frac{2}{3}$

Answer (2)

Sol. $P(A \cap B) = \frac{1}{10}$

$P(A/B) + P(B/A) = \frac{7}{12}$

and $P(A/B) \cdot P(B/A) = \frac{1}{12}$

$$\frac{P(A \cup B)}{P(B)} \cdot \frac{P(A \cap B)}{P(A)} = \frac{1}{12}$$

$$\Rightarrow \quad P(A) \cdot P(B) = 12\left(\frac{1}{12}\right)^2 = \frac{12}{100}$$

$$P(A \cap B)\left[\frac{1}{P(A)} + \frac{1}{P(B)}\right] = \frac{7}{12}$$

$$P(A) + P(B) = \frac{7}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{10} - \frac{1}{10} = \frac{6}{10}$$

$$\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})} = \frac{P(\overline{A \cap B})}{P(\overline{A \cup B})} = \frac{1 - P(A \cap B)}{1 - P(A \cup B)}$$

$$\frac{1 - \frac{1}{10}}{1 - \frac{6}{10}} = \frac{\frac{9}{10}}{\frac{4}{10}} = \frac{9}{4}$$

8. Number of terms in an arithmetic progression is $2n$. Sum of terms occurring at even places is 40 and sum of terms occurring at odd places is 55. If the first term exceeds the last term by 27, then n equals to

(1) 3 (2) 5

(3) 7 (4) 4

Answer (2)

Sol. Let the *AP* be

$a, a + d, a + 2d, \ldots, a + (2n - 1)d$

Now given that

$(a + d) + (a + 3d) + \ldots + (a + (2n - 1)d) = 40$

$na + n^2d = 40$ …(1)

Also $a + (a + 2d) + (a + 4d) + \ldots + (a + (2n - 2)d) = 55$

$na + dn(n - 1) = 55$ …(2)

Also $a - (a + (2n - 1)d) = 27$

$-(2n - 1)d = 27$ …(3)

$$d = \frac{-27}{2n - 1}$$

(2) – (1)

$dn(n - 1) - n^2d = 15$

$d[n^2 - n - n^2] = 15$

$$\left(\frac{-27}{2n - 1}\right)(-n) = 15$$

$27n = 30n - 15$

$15 = 3n$

$n = 5$

9. The perpendicular distance of point $P(3, 4, 5)$ from the line

$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(4\hat{i} - \hat{j} + 5\hat{k})$ is

(1) $\sqrt{\frac{19}{42}}$ (2) $\sqrt{\frac{19}{21}}$

(3) $\sqrt{\frac{42}{19}}$ (4) $\sqrt{\frac{21}{19}}$

Answer (1)

Sol. $P(3, 4, 5)$

L: $\vec{r} = \vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(4\hat{i} - \hat{j} + 5\hat{k})$

Any point on L can be. $A(2 + 4\lambda, -1 - \lambda, 1 + 5\lambda)$

Now $\overrightarrow{AP} \cdot (4\hat{i} - \hat{j} + 5\hat{k}) = 0$

$((4\lambda - 1)\hat{i} - (\lambda + 5)\hat{j} + (5\lambda - 4)\hat{k}) \cdot (4\hat{i} - \hat{j} + 5\hat{k}) = 0$

$16\lambda \div 4 + \lambda + 5 + 25\lambda - 20 = 0$

$42\lambda = 19$

$$\lambda = \frac{19}{42}$$

Now $|AP| = \sqrt{(4\lambda - 1)^2 + (\lambda + 5)^2 + (5\lambda + 4)^2}$

$$= \sqrt{\frac{19}{42}}$$

10. In the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, where α, β, γ and δ are the coefficient of x, x^3, x^5 and x^7 respectively. If $\alpha u - \beta v = 18$, $\gamma u + \delta v = 20$ then $u + v$ equal to.

(1) $\frac{-14}{15}$ (2) $\frac{-13}{15}$

(3) $\frac{-3}{5}$ (4) $\frac{-2}{3}$

Answer (1)

Sol. $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$$\left[\int_0^3 (x)^5+\int_1^5 x^4\sqrt{x^3-1}+\int_2^5 x^3\left(\sqrt{x^3-1}\right)^2+.....\right]+$$

$$\left[\int_0^5 x^5-\int_1^5 x^4\sqrt{x^3-1}+\int_2^5 x^3\left(\sqrt{x^3-1}\right)^2+.....\right]$$

$$= 2\left[x^5+\int_2^5 x^3(x^3-1)+\int_4^5 x(x^3-1)^2\right]$$

$= 2[x^5 + 10x^6 - 10x^3 + 5x(x^6 + 1 - 2x^3)]$

$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$

Coefficient of $x = 10 = \alpha$

Coefficient of $x^3 = -20 = \beta$

Coefficient of $x^5 = 2 = \gamma$

Coefficient of $x^7 = 10 = \delta$

$10u + 20v = 18$ ...(i)

$2u + 10v = 20$

$u = \frac{-11}{3}$

$v = \frac{41}{15}$

$u + v = \frac{41}{15} - \frac{11}{3}$

$= \frac{-14}{15}$

11. Let $A(6, 8)$, $B(10\cos\alpha, -10\sin\alpha)$ and $C(-10\sin\alpha, -10\cos\alpha)$ be 3 points and if orthocentre of the triangle ABC is (0, 9) then $100\sin^2\alpha$ is equal to

(1) $\frac{25}{4}$ (2) 25

(3) $\frac{15}{4}$ (4) $\frac{5}{4}$

Answer (1)

Sol. Notice, origin is equidistance form A, B and C

⇒ (0, 0) is circumcentre

Since centroid divides orthocentre and circumcentre in 2 : 1 ratio.

$$\Rightarrow \frac{6+10\cos\alpha+(-10\sin\alpha)}{3}=\frac{2(0)+1(0)}{3}=0$$

⇒ $15\sin\alpha - 10\cos\alpha = 6$

Alos $\frac{8-10\sin\alpha-10\cos\alpha}{3}=\frac{2(0)+1(9)}{3}=3$

⇒ $8 - 10\sin\alpha - 10\cos\alpha = 9$

$10(\sin\alpha + \cos\alpha) = -1$

$10(\sin\alpha - \cos\alpha) = -6$

$20\sin\alpha = 5$

$10\sin\alpha = \frac{5}{2}$

$100\sin^2\alpha = \frac{25}{4}$

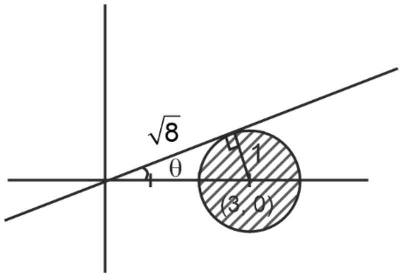
12. If z be a complex number such that $|z - 3| \le 1$, then the equation of line with largest slope passing through origin and z

(1) $x - 2\sqrt{2}y = 0$ (2) $x + 2\sqrt{2}y = 0$

(3) $2\sqrt{2}x + y = 0$ (4) $2\sqrt{2}x - y = 0$

Answer (1)

Sol.



$\tan\theta = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

Therefore, equation of line with maximum slope is

$(y-0) = \frac{1}{2\sqrt{2}}(x-0)$

$\Rightarrow \quad y = \frac{x}{2\sqrt{2}}$

13. A relation R is defined on set A, $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 3)\}$. Elements are added such that R becomes reflexive and transitive but not symmetric. Find the number of such relations.

(1) 3

(2) 4

(3) 2

(4) 9

Answer (1)

Sol. Transitivity

$(1, 2) \in R$, $(2, 3) \Rightarrow (1, 3) \in R$

$(1, 1), (2, 2), (3, 3) \in R$

$(2, 1) \qquad (3, 2) \qquad (3, 1)$

$(3, 1)$ cannot be taken.

1. (2, 1) taken and (3, 2) not taken.
2. (3, 2) taken and (2, 1) not taken.
3. Both not taken.

Therefore 3 relations are possible.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 21. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. If $\lambda\vec{a} + 3\vec{b}$ and $2\vec{a} + \lambda\vec{b}$ are perpendicular to each other, then the product of all possible values of λ is _____

Answer (6)

Sol. $|\vec{a}| = 1, |\vec{b}| = 1$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{3} = \frac{1}{2}$

$\lambda\vec{a} + 3\vec{b}$ and $2\vec{a} + \lambda\vec{b}$ are perpendicular

$\Rightarrow \quad (\lambda\vec{a} + 3\vec{b})\cdot(2\vec{a} + \lambda\vec{b}) = 0$

$\Rightarrow \quad 2\lambda + 3\lambda + (\lambda^2 + 6)\vec{a}\cdot\vec{b} = 0$

$\Rightarrow \quad 5\lambda + (\lambda^2 + 6)\left(\frac{1}{2}\right) = 0$

$\Rightarrow \quad 10\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \quad \lambda^2 + 10\lambda + 6 = 0$

Product of possible values of $\lambda = 6$

22. If A is the 3×3 matrix of order 3×3, such that $\det(A) = \frac{1}{2}$, $\text{tr}(A) = 10$ and B be another matrix of order 3×3 and defined as $B = \text{adj}(\text{adj}(2A))$, then $\det(B) + \text{tr}(B)$ is equal to (where tr(A) denotes trace of matrix A)

Answer (336)

Sol. $B = \text{adj}(\text{adj}2A)$

$B = |2A|^{n-2}(2A)$, [Using $\text{adj}(\text{adj } P) = |P|^{n-2}\cdot P$],

for $n = 3$

$= |2A|(2A)$

$= 2^3|A|(2A)$

$= 8 \times \frac{1}{2}(2A)$

$= 4(2A)$

$B = 8A$

$|B| = |8A|$

$= 8^3|A|$

$|B| = 8^3 \times \frac{1}{2} = 256$

$B = 8A$

[each element is multiplied 8 times]

$\text{tr}(B) = 8\text{tr}(A)$

$= 80$

$|B| + \text{tr}(B) = 256 + 80$

$= 336$

23. Consider two curves $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity e_1 and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ with eccentricity e_2. If $\frac{e_1}{e_2} = \frac{1}{3}$ and distance between foci of both curves is $2\sqrt{3}$ and $a - A = 4$, then the sum of lengths of latus rectum of both curves is

Answer (12)

Sol. Since, distance between foci

for $E_1 = 2ae$, for $E_2 = 2Ae_2$

$\Rightarrow\ 2ae_1 = 2\sqrt{3} = 2Ae_2$

$\Rightarrow\ ae_1 = Ae_2$

$\Rightarrow\ \frac{e_1}{e_2} = \frac{A}{a} = \frac{1}{3} \Rightarrow a = 3A$

Also $a - A = 4 \Rightarrow A = 2,\ a = 6$

Now, $2(6)e_1 = 2\sqrt{3} \Rightarrow e_1 = \frac{\sqrt{3}}{6}$

$2(2)e_2 = 2\sqrt{3} \Rightarrow e_2 = \frac{\sqrt{3}}{2}$

$e_1^2 = \frac{1-b^2}{a^2} = \frac{3}{36} = \frac{1-b^2}{36} \Rightarrow \frac{b^2}{36} = \frac{33}{36}$

$b^2 = 33$

$e_2^2 = \frac{1-B^2}{A^2} = \frac{3}{4} = \frac{1-B^2}{4} \Rightarrow B^2 = 1$

Length of latus rectum of $E_1 = \frac{2b^2}{a}$

Length of latus rectum of $E_2 = \frac{2B^2}{A}$

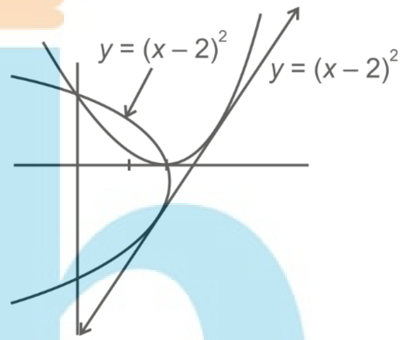
$\Rightarrow$ Sum of lengths of latus rectum

$= \frac{2(33)}{6} + \frac{2(1)}{2} = 11 + 1 = 12$

24. The number of maximum number of common tangents to the curves $y = (x - 2)^2$ and $y^2 = 16 - 8x$ is

Answer (1)

Sol.



Clearly, one tangent is possible. Based on the graphs of these parabola.

25. Let $P(10, -2, -1)$ and Q be the point of perpendicular drawn from point $R(1, 7, 6)$ on the line joining the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length PQ is

Answer (13)

Sol. $L_1 : \frac{x-2}{8} = \frac{y+5}{-12} = \frac{z-11}{16}$

$L_1 : \frac{x-2}{2} = \frac{y+5}{-3} = \frac{z-11}{4}$

Avg point of L_1 be $A(2\lambda + 2, -3\lambda - 5, 4\lambda + 11)$

$R(1, 7, 6)$

$RA.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 0$

$\left((2\lambda + 1)\hat{i} + (-3\lambda - 12)\hat{j} + (4\lambda + 5)\hat{k} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 0$

$4\lambda + 2 + 9\lambda + 36 + 16\lambda + 20 = 0$

$29\lambda = -58$

$\lambda = -2$

$\therefore$ Foot of perpendicular = (–2, 1, 3)Q

$PQ = \sqrt{(10 + 2)^2 + (1 + 2)^2 + (3 + 1)^2}$

$= \sqrt{144 + 9 + 16}$

= 13